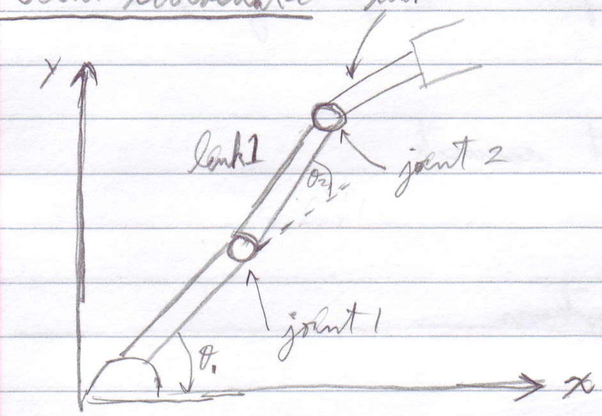
Engi-5557 Introduction to Robotics — Monday January 8, 2007

4-5 Assignments, Lab: 15%, Midterm: 25%, Final: 60% } Course outline on website

Robot Manipulators

Joint Coordinates

eg



forward kinematics (FK)

$$(\theta_1, \theta_2) \longrightarrow (x, y)$$

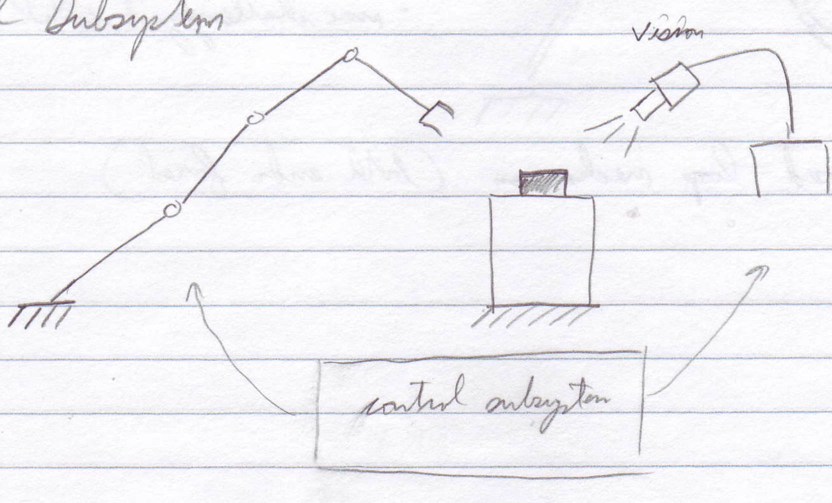
inverse kinematics (IK) (much more complex

i.e. soln. might not exist (not possible to reach by robot arm)

soln. not always unique (point can be reached in different ways.

- Robotic System: Composed of 3 distinct integrated sub-systems.

1. Motion Subsystem ✓
2. Recognition Subsystem (Vision and Sensing localization)
3. Control Subsystem ✓



Class of Robot Manipulators

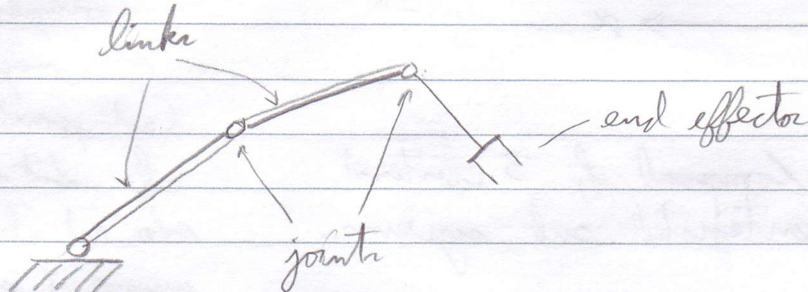
1. **Power Source** (to control joints)

a. Electric: AC, DC motors based actuators
used for accurate assembly

b. Hydraulic: fast in response, produce large torques, not very accurate

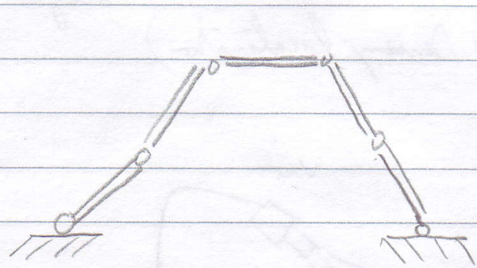
c. pneumatic: cheaper, but not accurate

Mechanics of Robotic Systems



industrial applications

open-loop mechanism (one-end fixed)

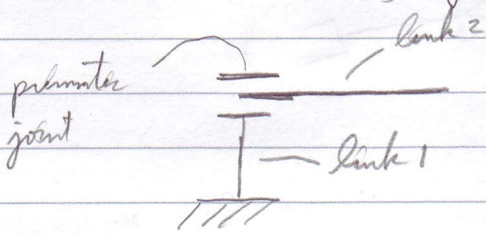


- more challenging to control

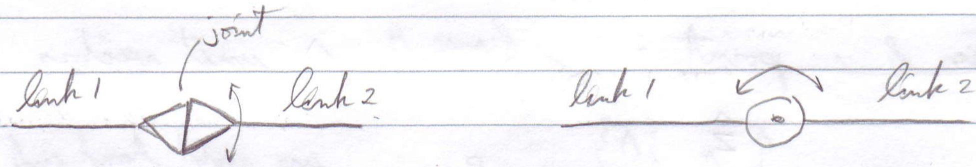
closed-loop mechanism (both ends fixed)

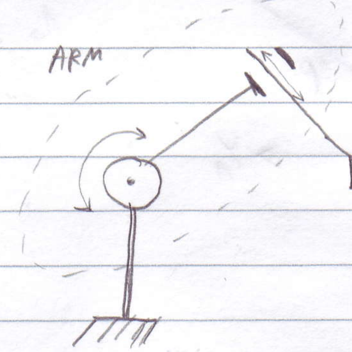
Typically we use 2 types of joints

1. **Prismatic (translation) joint** : joint is where 2 links are connected



2. **Revolute (Rotation) joint** :





combination of joints

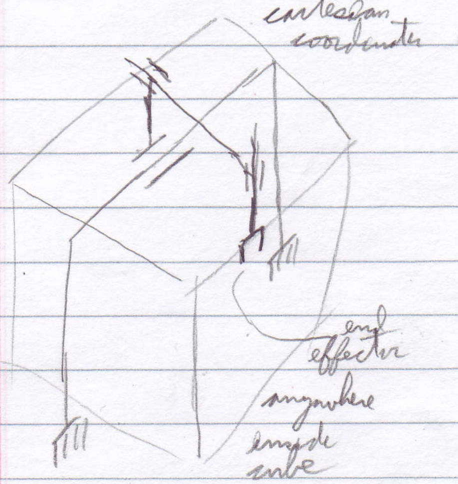
type of joint determines the workspace

Manipulator is divided into 3 ::
- arm portion
- wrist portion (3 rotations)
- hand portion

joint configuration examples

(i) 3 translational

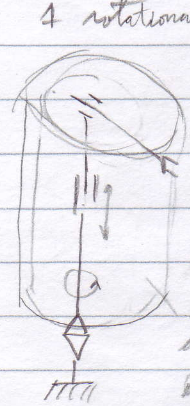
cartesian coordinates



end effector anywhere inside cube

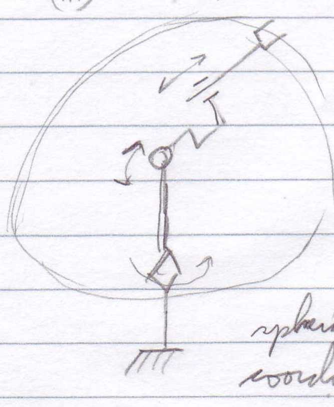
(ii) 2 translation joints
1 rotational joint

cylindrical coordinates



end effector can reach any point in cylinder

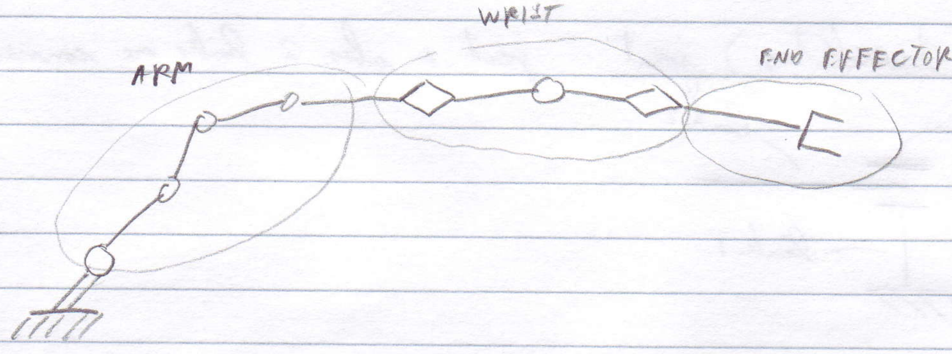
(iii) 1 trans, 2 rot



reach any point in sphere

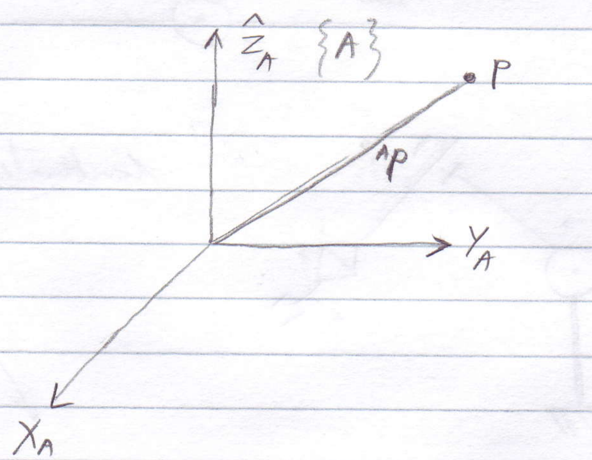
spherical coordinates

to get 6 degrees of freedom



• Rigid Motion and Homogeneous Transformations

<u>Description of a point</u> : frame A ^ unit vectors

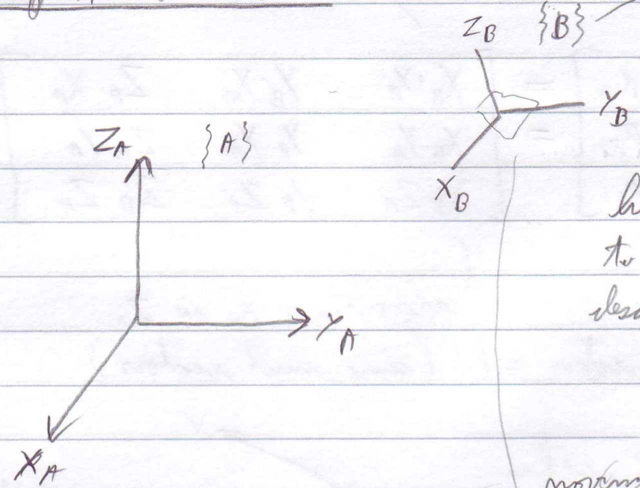


use right hand rule to establish axes

$$^A P = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

vector

Designation of an orientation



— frame B

all unit vector

how to describe → orientation of frame B WRT frame A

moving frame

fixed reference frame

need coordinates of X_B in frame {A}

3 points
3 points
3 points } form a rotation matrix

PROJECTION OF X.

$$ {}^{A}X_B = \begin{bmatrix} R_{11} \\ R_{21} \\ R_{31} \end{bmatrix} \qquad {}^{A}Y_B = \begin{bmatrix} R_{12} \\ R_{22} \\ R_{32} \end{bmatrix} \qquad {}^{A}Z_B = \begin{bmatrix} R_{13} \\ R_{23} \\ R_{33} \end{bmatrix} $$

components of each vector are the projection of each vector onto any direction of the reference frame A

Connect all the components to form the rotation matrix.

- the rotation matrix of frame {B} WRT frame {A} is =

$$ {}^{A}_{B}R = \left[{}^{A}X_B \;\; {}^{A}Y_B \;\; {}^{A}Z_B \right] = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} $$

$$ = $$

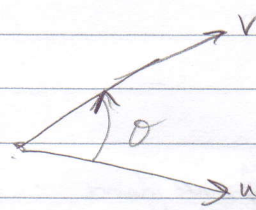
dot product $\therefore$ also called the cosine matrix since all components are cosines

$$
{}^{A}_{B}R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{bmatrix} X_B \cdot X_A & Y_B \cdot X_A & Z_B\, X_A \\ X_B\, Y_A & Y_B\, Y_A & Z_B\, Y_A \\ X_B\, Z_A & Y_B\, Z_A & Z_B\, Z_A \end{bmatrix}
$$

PROJECTION OF X_B ON Z_A

magnitude of all the vectors $= 1$ (since unit vectors)

eg Dot product of 2 vectors



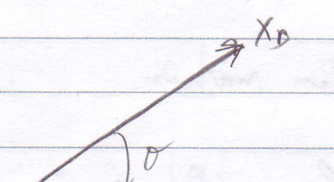
$u \cdot v = \|u\| \, |v| \cos\theta$

$$
\begin{aligned}
|X_A| &= |X_B| = 1 \\
|Y_A| &= |Y_B| = 1 \\
|Z_A| &= |Z_B| = 1
\end{aligned}
$$

$u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\qquad x_1 y_1 + x_2 y_2 = u \cdot v$ $\qquad$ dot product of 2 vectors = scalar

$v = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

eg.



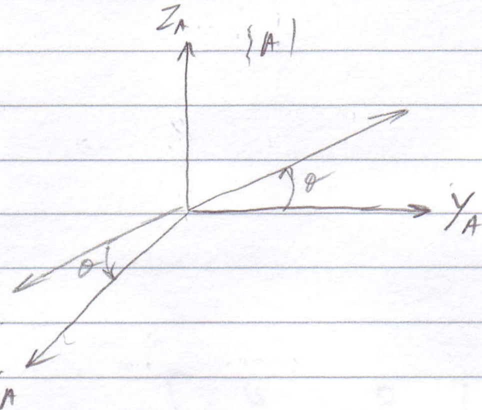
projection of X_B onto X_A

$R_{11} =$

$r_{11} = \cos\theta$ $\qquad$ since $|X_A| = |X_B| = 1$

$\left(r_{11} = |X_B| \cdot |X_A| \cos\theta \right) =$

1. Rotation about Z axis



frame B is ontop of frame A
(frame A is fixed)

Rotation is about Z_A

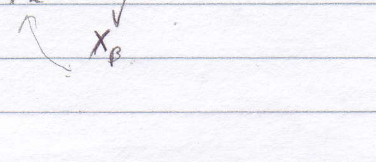
frame B WRT frame A about Z_A axis

$$^A_B R = R_{Z_A}(\theta) = \begin{bmatrix} \cos\theta & ? & 0 \\ ? & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

last row 001 } because have
last column 001 } rotation about Z axis

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

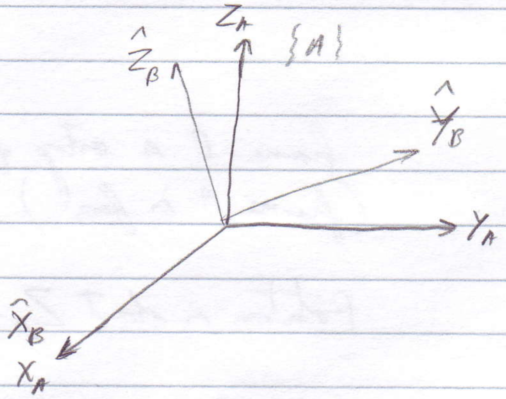
2. Rotation about Y axis : rotate frame B WRT Y axis



Rotation matrix {B} WRT {A}

$$^A_B R_{Y_A}(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

3. Rotation about X-axis:



$$ {}^{A}_{B}R = R_{X_A}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} $$